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I VS I-DOT

The recurrent dilemma between using a "current probe" (a self-integrating I-dot sensor) and using an I-dot sensor can often be resolved by understanding the similarities and differences of the two types of sensors. Both may be thought of as toroidal arrays of B-dot sensors, as indicated in Figure 1. A time variant current through the aperture (I-dot) gives rise to a time variant magnetic field around the current (B-dot) according to Ampere's Law:

$$B = \frac{\mu I}{2\pi r} \rightarrow \frac{dB}{dt} = \frac{\mu}{2\pi r}\frac{dI}{dt} \text{ or } \dot{B} = \frac{\mu \dot{I}}{2\pi r} \qquad (1)$$

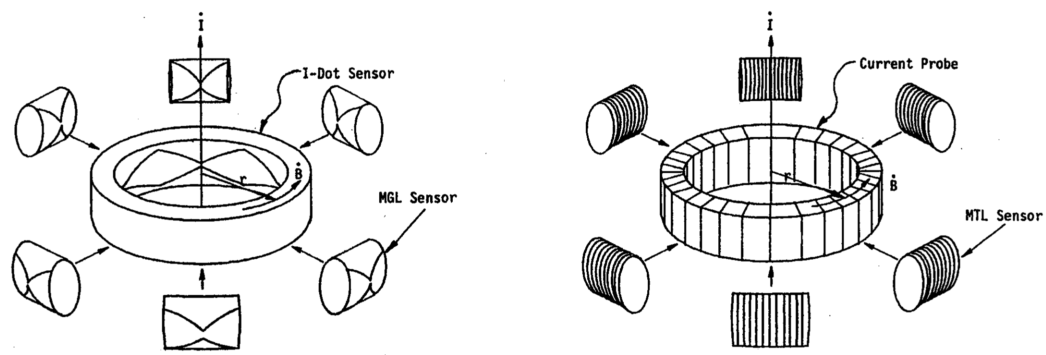

Figure 1. Current Sensors as combined B-Dot Loops

The flux of this field through N loops of a rectangular toroid such as shown in Figure 2 is

$$\phi B = \oint_{loop} \vec{B}\cdot d\vec{A} = \int_{r1}^{r2} \frac{N\mu I}{2\pi r} h dr = \frac{N\mu I h}{2\pi}\int_{r1}^{r2}\frac{dr}{r} = \frac{N\mu I h}{2\pi} \ln\frac{r2}{r1} \qquad (2)$$

Applying Faraday's Law to the loop,

$$\oint \vec{E}\cdot d\vec{l} = \frac{d}{dt}\phi B = -\frac{\mu N}{2\pi} h \ln\frac{r2}{r1}\dot{I}$$

or, with $V_o = \oint \vec{E}\cdot d\vec{l}$ and $M = \frac{\mu N}{2\pi} h \ln \frac{r2}{r1}$ (the Mutual Inductance),

$$V_o = M\dot{I} \qquad (3)$$

Let the toroid be separated into discrete segments as in Figure 1 and take B as the average value of B(r) across the equivalent area A_{eq} of the loops. Then $\phi B = B A_{eq}$ and Faraday's Law yields

$$V_o = A_{eq}\dot{B} \qquad (4)$$

Combining Equations (3) and (4),

$$V_o = M\dot{I} = A_{eq}\dot{B} \rightarrow M = \frac{A_{eq}B}{I}$$

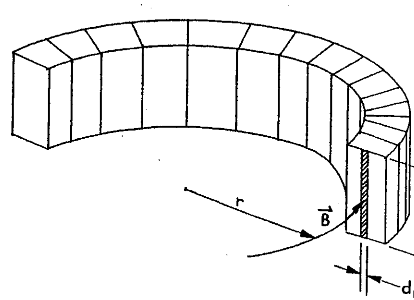

Figure 2. Magnetic Flux through Toroid Loops

From Equation (1), $B/I = \mu/2\pi r$, so

$$M = \frac{\mu}{2\pi r} A_{eq}$$

Thus an I-dot sensor of mutual inductance M may be thought of as a set of B-dot sensors of equivalent area A_{eq} assembled into a toroid of radius r. This could be implemented by installing six or eight small MGL or MTL B-dot sensors around the girth of a missle body to measure the total current flowing through the missile.

The current probe cannot properly be called a current sensor because it does not respond to direct current (for which I-dot and the associated B-dot = 0). It is designed to sense and integrate the first time derivative of the current, or, more precisely, the magnetic field associated with the current being measured (to operate in the self-integrating mode), the way multi-turn loop (MTL) B-dot sensors operate. The I-dot sensor is properly called a sensor because it is designed to only sense the first time derivative of the magnetic field associated with the current being measured (to operate in the differentiating mode), the way multi-gap loop (MGL) B-dot sensors operate. Current probes and I-dot sensors both sense the same thing, but as componets of a measurement system, they operate in different modes. The mode in which an I-dot or B-dot sensor operates depends on the frequency of the current producing the field and the inductance and impedance of the loop or loops. We can understand this more clearly by considering the operation of a multi-turn B-dot sensing loop. If we break the loop and attach a cable and readout device of impedance Z to the gap, we can draw the equivalent circuit, as shown of Figure 3. Here, we have a voltage source whose function is $V_0 = \dot{B} A_{eq}$ in series with the self-inductance, L, of the loop and the load impedance, Z, of the cable.

Adding voltages around the circuit, we have

$$\dot{B} A_{eq} = L\dot{i} + iZ, \quad (5)$$

where i is the current in the loop. With $L/Z = \tau$ (the time constant of the sensor) and $V_0 = iZ$, Equation (5) becomes

$$\dot{B} A_{eq} = \tau\dot{V}_0 + V_0 \quad (5a)$$

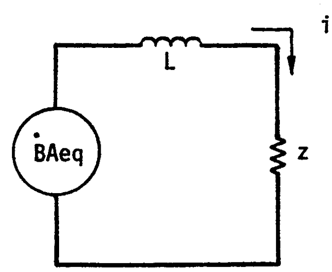

Figure 3. Equivalent Circuit of B-dot Sensor

The differential Equation (5) can be solved conveniently by use of Laplace transforms. The Laplace transform of Equation (5) is

$$sB(s)A_{eq} = si(s)L + i(s)Z$$

where $s = j\omega$ and $|s| = |j\omega| = \omega$. Solving for i,

$$i(s) = \frac{sB(s)\,A_{eq}}{sL + Z} \quad (6)$$

<u>Low Frequency Response</u>

At low frequencies, the inductive reactance is low and most of the voltage in the loop appears across the cable impedance. That is, for $|sL| = \omega L \ll Z$, we neglect ωL in the denominator for Equation (6), leaving

$$i(s) = \frac{sB(s)A_{eq}}{Z}$$

Transforming back to the time domain,

$$i(t) = \frac{\dot{B}(t)\, A_{eq}}{Z}$$

or (7)

$$V_o = iZ = \dot{B}\, A_{eq}$$

This is the low frequency transform function of the sensor.

High Frequency Response

At high frequencies, the inductive reactance is high. Most of the voltage in the loop appears across the inductance, and we can neglect the voltage across the load. That is, for $|sL| = \omega L >> Z$, we neglect Z in the denominator of Equation (6), leaving

$$i(s) = \frac{sB(s)\, A_{eq}}{sL} = \frac{B(s)\, A_{eq}}{L}$$

Transforming back to the time domain, we have

$$i(t) = \frac{B(t)\, A_{eq}}{L}$$

or, using $V_o = iZ$,

$$V_o = B\, A_{eq} \frac{Z}{L} = \frac{B\, A_{eq}}{\tau} \qquad (8)$$

where τ is the time constant of the sensor (= L/Z).

Equation (8) is the high frequency transfer function of the sensor.

The Transitional Mode

At frequencies where $\omega L \approx Z$, neither Equation (7) nor (8) give the correct transfer function by itself. Equation (5) must be used.

The relationship between the high and low frequency transfer functions is shown in Figure 4, which is a plot of Equation (6). If $\omega < \omega_1$, the sensor is said to be operating in the differentiating mode, i.e., the output is proportional to the first time derivative of B. If $\omega > \omega_2$, the sensor is said to be operating in the self-integrating mode, i.e., while the "sensed" voltage is proportional to $\dot{B}$, the output voltage is proportional to B, the antiderivative or integral of $\dot{B}$. If $\omega_1 < \omega < \omega_2$, the sensor is operating in the transitional mode, i.e., the output is given implicitly by Equation (5).

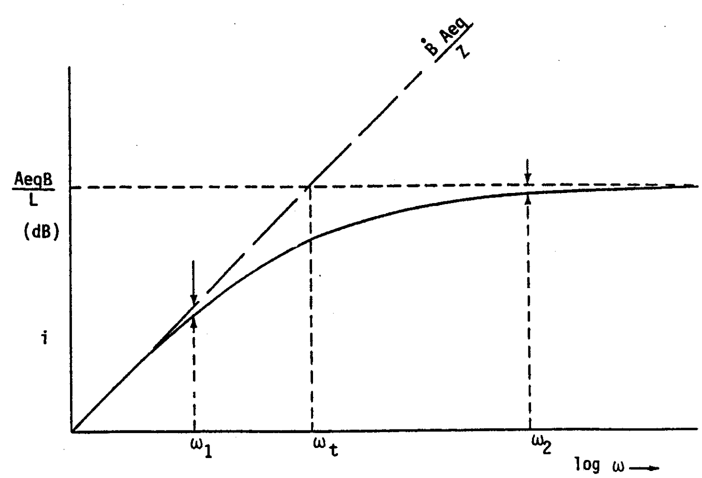

Figure 4. Frequency Response of MTL Sensor

The Time Domain

To better understand these three cases, it is helpful to consider the case in the time domain where B is a pulse of variable risetime (and duration). Figure 5 shows the three modes of operation. In Figure 5a, we have a pulse with a long risetime (low frequency). The sensor operates in the differentiating mode. In Figure 5b, the pulse has a short risetime (high frequency). The sensor is operating in the self-integrating mode and the output is proportional to the input. The pulse in Figure 5c has a risetime on the order of the time constant of the sensor, i.e.,

$$t_r \quad \left(\approx \frac{1}{\omega}\right) = \frac{L}{Z}$$

or

$$Z \approx \omega L.$$

The sensor gives a correct peak value using Equation (8), but the value decays exponentially according to $e^{-t/\tau}$. The transitional frequency is defined as the frequency where the inductive reactance equals the load impedance:

$$\omega L = Z$$

$$2\pi f L = Z$$

or

$$2\pi f_T = \frac{Z}{L} = \frac{1}{\tau}$$

$$f_T = \frac{1}{2\pi\tau} \qquad (9)$$

Frequency Ranges

The frequency ranges for the differentiating and self-integrating modes are determined by choosing an allowable error, ε, in the output voltage due to responses in the opposite mode. By considering the vector magnitudes of the terms of Equation (5), it is easily shown that

$$\varepsilon_1 = \frac{\dot{B} A_{eq} - V_o}{\dot{B} A_{eq}} = \frac{\sqrt{a^2+1} - 1}{\sqrt{a^2+1}} \qquad (10)$$

and that

$$\varepsilon_2 = \frac{\dfrac{A_{eq} B}{\tau} - V_o}{\dfrac{A_{eq} B}{\tau}} = \frac{\sqrt{a^2+1} - a}{\sqrt{a^2+1}} \qquad (11)$$

where

$a = f/f_T$

ε_1 = error in low frequency transfer function

ε_2 = error in high frequency transfer function.

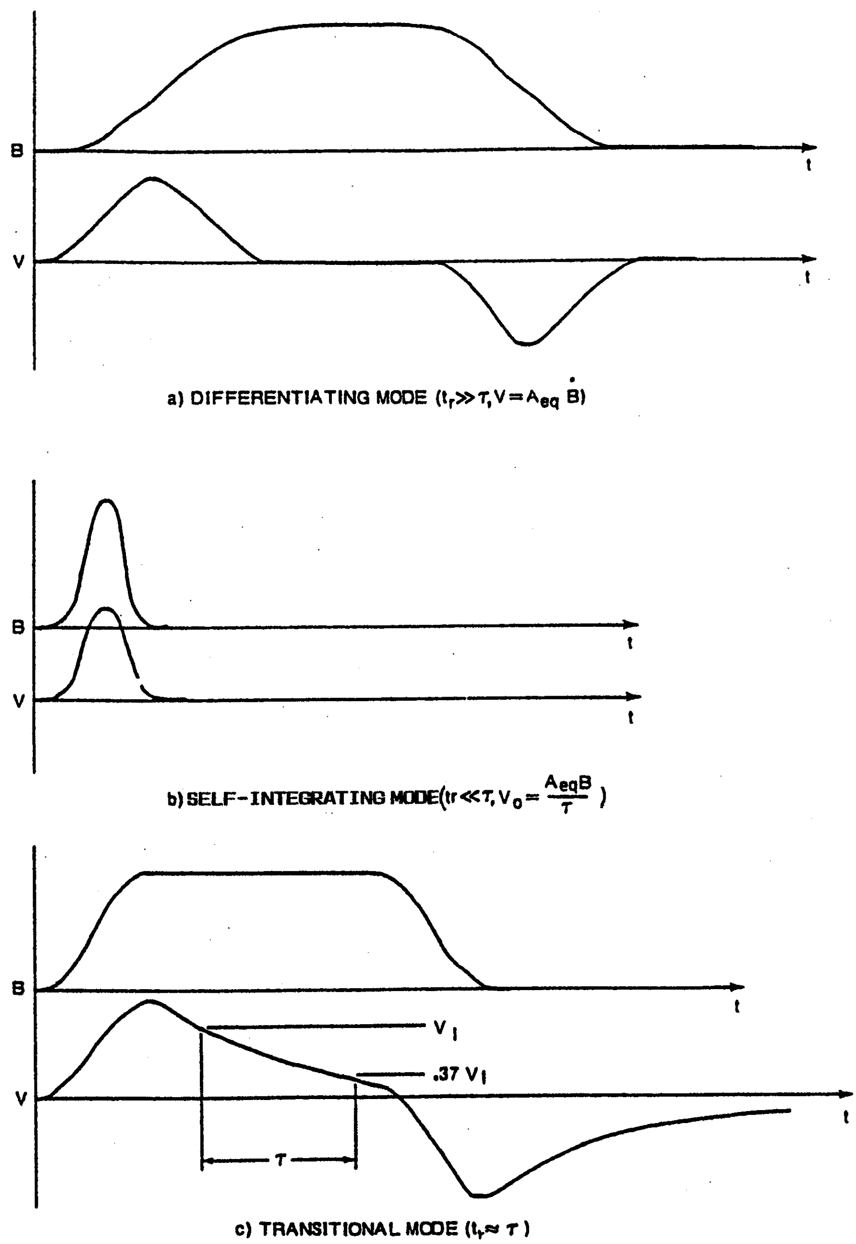

Figure 5. Time Domain Response of MTL Sensor

Equations (10) and (11) show that ε_1 increases with increasing frequency (increases with a) and that ε_2 decreases with increasing frequency (decreases with a), as would be expected. This is also shown implicitly in Figure 4. Several values of a, ε_1 and ε_2 are shown in Table 1.

TABLE 1. ERRORS FOR VARIOUS VALUES OF a

a	0.1	0.143 (1/7)	0.25	0.33	0.5	1	2	3	4	7	10
ε_1	0.005	0.010	0.030	0.050	0.106	0.293					
ε_2						0.293	0.106	0.050	0.030	0.010	0.005

Note the logarithmic symmetry of ε_1 and ε_2 about a = 1. If f_1 (the upper cutoff frequency for the differential mode) is 0.1 f_T (a = 0.1) and if f_2 (the lower cutoff frequency for the self-integrating mode) is 10 f_T (a = 10), then $\varepsilon_1 = \varepsilon_2 = 0.005$. In other words, if we limit the upper frequency for the differential mode to 1/10 f and the lower frequency for the self-integrating mode to $10f_T$, the maximum error in each case is 1/2%. If we want 1/2% accuracy across the spectrum, we may use Equation (7) below 0/1 f_T; we must use Equation (5) between 0.1 f_T and 10 f_T; and we may use Equation (8) above 10 f_T.

The output of the sensor used in either mode is down .293 or 3 dB when $f = f_T$ (a = 1). This means that the upper 3 dB frequency for the differentiating mode is the lower 3 dB frequency for the self-integrating mode. Thus a sensor is an "I-dot sensor" or a "current probe" (an I sensor) depending on the value of a. An "I-dot sensor" is designed for a high transition frequency, while a "current probe" is designed for a low transition frequency. Recall that the transitional frequency is that for which the inductive reactance is equal to the load impedance (Equation 9):

$$f_T = \frac{Z}{2\pi L}$$

This means that the transition frequency of a sensor can be raised by reducing the inductance and vice versa. Inductance varies as the number and size of the turns, and the permeability of the medium within the turns. Accordingly, "current probes" usually have multiple turns and highly permeable cores to increase the inductance, while I-dot sensors usually have a single turn (a toroidal conductive surface with a single gap) and an air core to provide low inductance.

Replacing the voltage source (A_{eq} B-dot) in the equivalent circuit of the B-dot sensor (Figure 3) with M I-dot gives the eqivalent circuit of an I-dot sensor. Equation 5 is then rewritten

$$M\dot{I} = L\dot{i} + Z\,i \tag{12}$$

Taking the Laplace transform as in Equation (6),

$$M\,sI(s) = L\,si(s) + Z\,i(s)$$

$$i(s) = \frac{M\,sI(s)}{sL + Z} \tag{13}$$

Recalling $|s| = |j\omega| = \omega$ and letting $\omega L << Z$ and $\omega L >> Z$ as in Equations (7) and (8), we obtain the low frequency (differentiating mode) and high frequency (self-integrating mode) transfer functions of the I-dot sensor:

Low Frequency:

$$i(s) = \frac{M\,sI(s)}{Z}$$

$$i(t) = \frac{M\dot{I}}{Z}$$

with $V_o = iZ$,

$$V_o = M\dot{I} \tag{14}$$

High Frequency:

$$i(s) = \frac{M\,sI(s)}{sL}$$

$$i(t) = \frac{M\,I}{L}$$

$$V_o = \frac{M\,I}{L} \tag{15}$$

Observing that the self-inductance L of a coil is the number of turns N times the mutual inductance M (L = NM) and defining the transfer impedance $Z_T = Z/N$, Equation (15) can be rewritten

$$V_o = Z_T\,I \tag{15a}$$

Equations (14) and (15a) are the familiar forms of the transfer functions of I-dot sensors and current probes (which are really self-integrating I-dot sensors).

Rise Time versus Frequency

Many of the electric and magnetic field sensors (including I-dot sensors) used today in CW testing were developed for EMP pulse testing applications. The performance of these sensors is often described in the time domain, eg, the frequency response may be stated as a risetime. It is useful to be able to transform sensor performance data between the time and frequency domains. The 10-90% risetime of a sensor is related to its frequency response by equating the initial slopes of the pulse output $V(t) = V_0\,(e^{-t/a\tau} - e^{-t/b\tau})$ to the CW output $V(t) = V_p \sin 2\pi ft$. The risetime associated with the pulse is .25 T, where T is the period of the sine function. The upper limit of the flat (0 db) frequency response of a sensor with a risetime of t_r may be calculated as follows:

$$t_r = .25\,T \Rightarrow \frac{1}{T} = \frac{.25}{t_r}$$

$$f_0 = \frac{1}{T} = \frac{.25}{t_r} \qquad (16)$$

The 10-90% risetime of a sensor cannot be less than the reciprocal of its maximum slewing or signal change rate, which is a measure of the maximum signal derivative the sensor can respond to without degrading the signal. Differentiating the sine function $V(t) = V_p \sin(2\pi ft)$ gives:

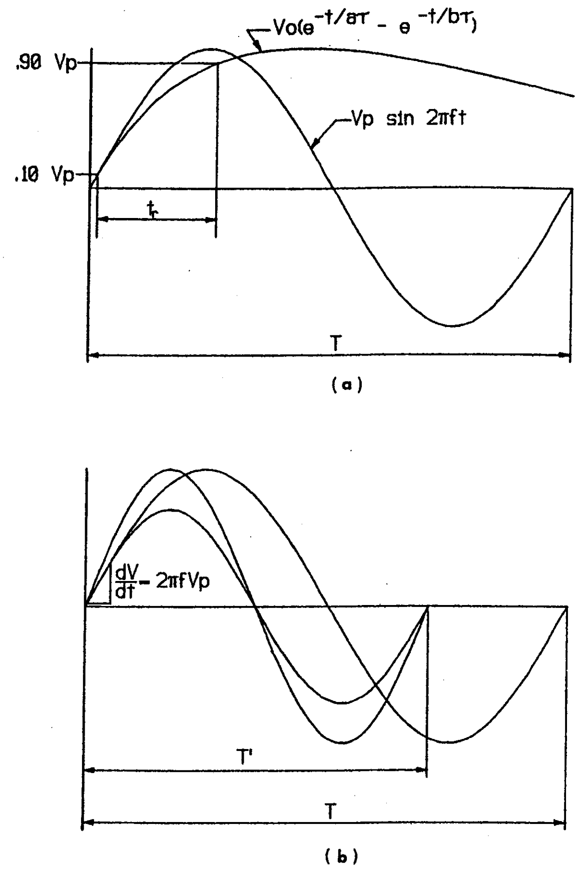

Figure 6. Pulse Risetime vs CW Frequency

$$\frac{dV}{dt} = 2\pi fV_p \cos 2\pi ft$$

This signal derivative is maximum when $\cos 2\pi ft = 1$ (when $t = 0$), and has magnitude $2\pi fV_p$. If we raise the frequency from f to f' (shorten the period from T to T') as shown in Figure 6b, the sensor will limit the amplitude of its output so that its maximum slew rate will not be exceeded:

$$\left|\frac{dV}{dt}\right|_{max} = 2\pi f'V_p' = 2\pi fV_p$$

then

$$f' = f\,\frac{V_p}{V_p'}$$

We may calculate the 3 db frequency by setting $V_p' = .707\,V_{p_0}$.

Then

$$f_3 = f_0\,\frac{V_{p_0}}{.707\,V_{p_0}} = 1.414\,f_0 = 1.414\,\frac{.25}{t_r}$$

$$f_3 = \frac{.35}{t_r} \qquad (17)$$

Pulsed Power Dissipation

It is often necessary to transform a specified cw power rating to the maximum pulse frequency a current probe can handle. Figure 7 compares one cycle of a sine wave of frequency f_r to a pulse in a series of pulses with repetition rate f_r. The energy in one cycle of a sine wave is

$$E_s = \int_0^T \frac{V^2}{R}\,dt = \frac{V_{cw}^2 T}{2R} = P_{cw}T$$

The energy in one pulse is

$$E_p = \int_0^T \frac{V^2}{R}\,dt = \frac{V_p^2 t_H}{R}$$

where t_H is the pulse width at $V = .5\ V_p$ (the "full width of half maximum amplitude")
Equating the two expressions, we have

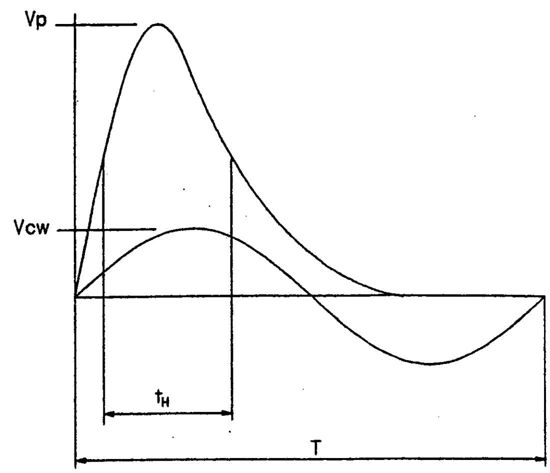

Figure 7. CW Power vs Pulsed Power

$$\frac{V_p^2 T_H}{R} = \frac{V_{cw}^2 T}{2R} \Rightarrow \frac{t_H}{T} = \frac{V_{cw}^2}{2V_p^2} \Rightarrow t_H f_r = \frac{P_{cw}R}{V_p^2} \qquad (18)$$

WHICH SENSOR TO USE

Informed consideration of sensitivity, bandwidth, signal processing and power and voltage limitations result in an optimal choice of sensor for each application. The characteristics of I-dot sensors and current probes in each of these areas are summarily compared in the following.

I-dot Sensors

The sensitivity of an I-dot sensor is determined entirely by its physical dimensions. It does not need calibration since the dimensions do not change over time. I-dot sensors are available with sensitivities ranging from .5 nonohenries (nH) for the PRODYN RID-240 to 10 nH for the PRODYN RID-270, and a much wider range could easily be provided if required. Risetimes for these sensors range from .17 to 1.8 ns, which corresponds to upper 3 dB frequencies ranging from about 190 MHz to 2 GHz. The wide range of output voltage (5 mV to 5 kV) and the differentiating mode of these sensors enable them to meet a wide range of measurement requirements (8×10^{-5} to 1.6×10^{3} A/ns). For CW measurements where $I(t) = I_p \sin 2\pi ft$, observe that

$$\dot{I} = 2\pi f I_p \cos 2\pi ft \Rightarrow \dot{I}_p = 2\pi f I_p$$

recalling Equation (14),

$$V_{o_p} = M \dot{I}_p = M2\pi f I_p \Rightarrow \dot{I}_p f = \frac{V_{o_p}}{2\pi M} \qquad (19)$$

This provides for peak currents ranging from .08 mA at 1 GHz (80 mA at 1 MHz or 80 A 1 KHz) to 1600 A at 1 GHz (1.6 x 10^6 A at 1 MHz or 1.6 x 10^9 A 1 KHz).

The lower frequency limit for an I-dot sensor is determined by the lowest $I_p f$ which provides a useable signal. For an RID-240 putting out 5 mV, Equation (19) gives

$$I_p f_{min} = \frac{5 \times 10^{-3} \text{ V}}{2\pi(5 \times 10^{-10} \text{ V sec/A})} = 1.59 \times 10^6 \text{ A/sec}$$

or 1.59 MHz @ 1 A, 1.59 KHz @ 1000 A, etc.

The upper frequency limit is $.35/t_r$, the sensor risetime, which depends on the relative magnitudes of its constituents t_{rL}, the risetime of the LZ equivalent circuit (Figure 3) and t_{rt}, the sensor loop propagation time or transit time. Using Equations (17) and (9), we may write

$$t_{rL} = \frac{.35}{f_T} = \frac{.35(2\pi L)}{Z} = \frac{.7\pi NM}{Z} \qquad (20)$$

and defining l_t as the distance around the sensing loop, we may write

$$t_{rt} = \frac{l_t}{c} \qquad (21)$$

The sensor risetime will be limited by the inductive component t_{rL}, if t_{rL} is longer than the transit time t_{rt}. If the transit time is longer than the inductive risetime, it will combine vectorially with the inductive risetime to limit the sensor risetime:

$$t_r = \begin{cases} \sqrt{t_{rL}^2 + t_{rt}^2}, & t_{rt} > t_{rL} \\ t_{rL}, & t_{rt} > t_{rL} \end{cases} \qquad (22)$$

for an RID 210, N = 2, M = 2 nH, Z = 100 ohm and l_t = 6.81 cm.

Then from Equation (20),

$$t_{rL} = \frac{.7\pi(2)2 \times 10^{-9} \text{ V sec/A}}{100 \text{ ohm}} = .088 \times 10^{-9} \text{ sec} = .088 \text{ ns}$$

and from Equation (21)

$$t_{rt} = \frac{6.81 \times 10^{-2}\text{m}}{3 \times 10^8 \text{ m/sec}} = 2.269 \times 10^{-10} \text{ sec} = .227 \text{ ns}$$

This sensor's risetime is limited by its transit time component and is also retarded by the inductive component. The resultant risetime, from Equation (22), is

$$t_r = \sqrt{(.088 \text{ ns})^2 + (.227 \text{ ns})^2} = .243 \text{ ns}$$

and the upper 3 dB frequency is, from Equation (17)

$$f_3 = \frac{.35}{.243 \times 10^{-9}\ \text{sec}} = 1.44 \times 10^9/\text{sec} = 1.44\ \text{GHz}$$

I-dot sensor voltage output is limited by standoff voltage at critical points in the sensor such as the connector or the gap connection. For most PRODYN I-dot sensors, the limitation is 5 kV differential and 2.5 kV to ground.

Current Probes

The sensitivity and bandwidth of current probes are not related the way they are for I-dot sensors. While the output voltage from an I-dot sensor is a linear function of the frequency of the current being sensed, the output voltage from a current probe is independent of frequency in its useful bandwidth. In fact, the useful bandwidth is defined as the frequency range for which the output voltage is independent of frequency, the theoretical lower limit being the transition frequency.

A current probe is a current transformer, the measured current flowing in a one turn primary "winding" and the signal current flowing in the N turn secondary winding. The sensitivity (the transfer impedance, Z_T) of a current probe depends on the number of turns N in the secondary winding and the load impedance of the secondary winding Z_L (usually a 50 ohm cable, which may be combined in parallel with an internal resistor to adjust the transfer impedance):

$$Z_T = \frac{Z_L}{N} \tag{23}$$

where

$$Z_L = \frac{R_i\ Z_c}{R_i + Z_c}$$

Current probes are available with transfer impedances from .03 ohm for the PRODYN I-125-3A to 5 ohms for a number of PRODYN models. The I-125-3A current probe can handle an output as high as 400 volts, so it can measure peak currents as high as 1.33×10^4 amps in its useable bandwidth (1 KHz to 100 MHz). The I-125-1B current probe has a transfer impedance of 5 ohms and a bandwidth of 45 KHz to 100 MHz. With a readout system capable of using a 5 mV signal, it could measure peak currents as small as 1 mA.

Current probe performance is limited by standoff voltage at critical points in the probe such as the connector or the internal shunt resistor, induction saturation in the core and resistive heating in the winding. Standoff voltage limits the output voltage to 2000 V in probes with no internal resistor and 400 V in probes with an internal resistor. Induction saturation limits the measured current and the time interval during which it can be applied. Resistive heating limits the power which can be dissipated in the winding and hence the magnitude of a cw current input or the amplitude-frequency product of a pulse train.

The Pros and Cons

The advantages of the I-dot sensor over the current probe are that it needs no periodic calibration, has a wider useful bandwidth, and a larger maximum output capability, which provides greater dynamic range. The disadvantage is the output must be integrated if I rather than I-dot is the measurand. The advantage of the current probe over the I-dot sensor is the pre-integrated signal: the disadvantages are narrower operating bandwidths, the inductive and thermal limitations and the calibration requirement.