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ELECTRIC & MAGNETIC FIELD SENSOR APPLICATION

(A WAY TO GET FROM MAXWELL'S EQUATIONS TO FIELD MEASUREMENTS)

Electric ("D-Dot") and magnetic ("B-Dot") field sensors, which were developed to measure Electro-Magnetic Pulse (EMP) phenomena, are now being used to measure lightning and other pulse and continuous wave EM phenomena. These new applications typically do not employ the same jargon the EMP practitioners use, so many potential users are having difficulty using the sensor "transfer function" to choose the appropriate size sensor.

The field sensor transfer function is a statement of the sensitivity of the sensor, giving the voltage output as a function of the flux of the electric or magnetic field through the sensor and the size (equivalent area) of the sensor. The sensitivity of the sensor is, in fact, its equivalent area. The development of the transfer functions of these sensors requires the following definitions:

$\mathbf{E}$, electric field strength, v/m
ϵ_o, permittivity constant, 8.85×10^{-12} farad/m or coul/v-m
$\mathbf{D}$, electric displacement, coul/m^2, ($= \epsilon_o \mathbf{E}$ in free space)
$\mathbf{H}$, magnetic field strength, amp/m
μ, permeability, weber/amp-m, ($= \mu_r \mu_o$)
μ_r = relative permeability, dimensionless
μ_o = permeability of free space, $4\pi \times 10^{-7}$ weber/amp-m or h/m
(1 weber = 1 v-sec and 1 weber/amp = 1 h)
$\mathbf{B}$, magnetic induction, weber/m^2 or v-sec/m^2 ($= \mu \mathbf{H}$)
y(1 weber/m^2 = 1 Tesla = 10^4 Gauss)
ϕ, Field flux

$\phi_E = \int \mathbf{E} \cdot d\mathbf{A}$, electric flux through a surface, v-m

$\phi_B = \int \mathbf{B} \cdot d\mathbf{A}$, magnetic flux through the surface enclosed by a loop, weber or v-sec

The transfer function for an electric field (D-Dot) sensor is developed by applying Gauss's law to a conductive element in, but electrically isolated from, a ground plane (see Figure 1). Let the element be enclosed in a closed (Gaussian) surface. Then:

$$\phi_E = \int \mathbf{E} \cdot d\mathbf{A} = q/\epsilon_o \qquad \text{(Gauss's Law)},$$

where q is the charge enclosed by the Gaussian surface. We generally arrange the sensor geometry so that the integral is easy to calculate, and we write

$$\int \mathbf{E} \cdot d\mathbf{A} = E\, A_{eq} \tag{1}$$

where A_{eq} is the equivalent area of the sensor. The equivalent area is defined by rewriting Equation (1):

$$A_{eq} = \frac{\int \mathbf{E} \cdot d\mathbf{A}}{E} \tag{2}$$

The transfer function for a magnetic field (B-Dot) sensor is developed by applying Faraday's law to the surface enclosed by a conductive loop (see Figure 2). Let the loop be interrupted by a voltage sensing gap. Then:

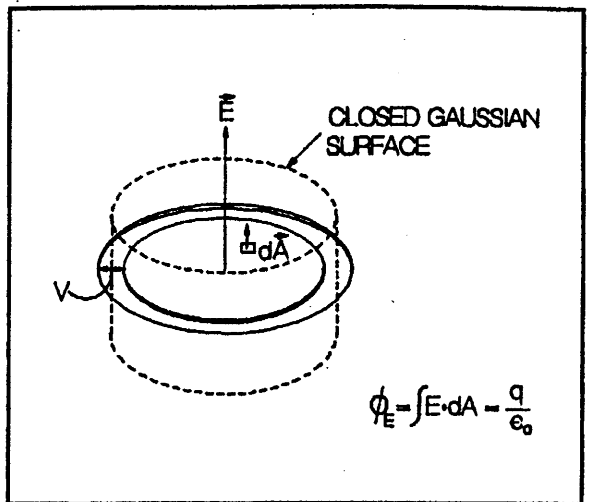


Figure 1. Conductive element in a ground plane

$$-\frac{d}{dt}\phi B = -\frac{d}{dt}\int \mathbf{B} \cdot d\mathbf{A} = \int \mathbf{E} \cdot d\mathbf{l} \quad \text{(Faradays's Law)},$$

where **E** is the electric field associated with the magnetic field and dl is a differential element of the loop. The integral on the right yields the voltage at the gap, V_o. As with the electric field sensor, we generally arrange the sensor geometry so that the integral on the left is easy to calculate, and we write

$$\int \mathbf{B} \cdot d\mathbf{A} = B\, A_{eq} \tag{3}$$

where A_{eq} is again the equivalent area of the sensor. The equivalent area is defined by rewriting Equation (3):

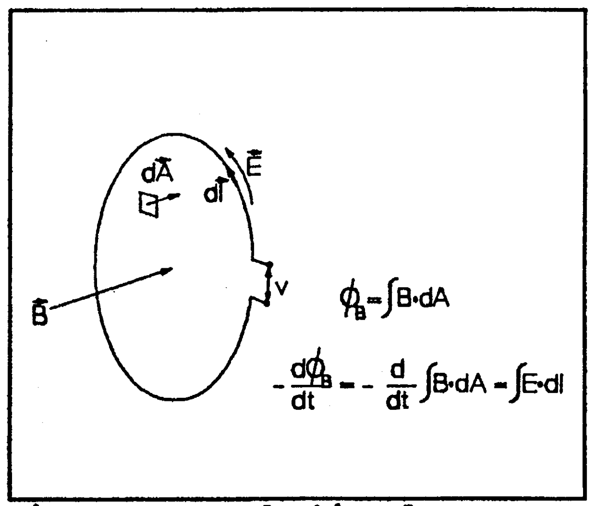


Figure 2. Conductive loop

$$A_{eq} = \frac{\int \mathbf{B} \cdot d\mathbf{A}}{B} \tag{4}$$

For both sensors, the equivalent area is a scalar constant which results from $\int \mathbf{F} \cdot d\mathbf{A}$, depending on the magnitude and direction of the field **F** (**E** or **B**) and the size (magnitude) and orientation (direction) of the sensing element. Once the geometry of a sensing element is determined, its equivalent area is defined, which in turn fixes its sensitivity to the field component along its axis. Since the sensitivity of the element is the equivalent area, which is constant, <u>periodic calibration is unnecessary.</u>

The term "sensor" is used to distinguish this class of instruments, which measure without transforming energy, from "transducers", which measure by transforming energy from one form to another (eg, mechanical to electrical as in a strain gage). The sensor is not encumbered by an energy transformation mechanism (electrical phenomena are sensed electrically), whereas the transducer depends on some transformation mechanism which usually has some nonlinearity and variability over time. These deviations from ideal performance make it necessary to calibrate transducers periodically.

Electric and magnetic field sensors measure the first time derivative of the field. Gauss's Law for the electric field sensor may be rewritten

$$E\ A_{eq} = q/\epsilon_o \qquad (5)$$

To measure the charge on the sensor element, we allow it to flow through an impedance, creating a current. This is expressed by differentiating Equation (5):

$$\frac{d}{dt}\ \epsilon_o\ E\ A_{eq} = \frac{dq}{dt} = i = \frac{V_o}{R}$$

where i is the current flowing through the transmission line to the measurement system, R is the impedance of the transmission line and V_o is the voltage across R. Using the definition $D = \epsilon_o E$ and $dD/dt = \epsilon_o dE/dt$, we write

$$\dot{D}\ A_{eq}\ R = V_o \qquad (6)$$

Equation (6) is the transfer function of the D-Dot sensor.

Using Equation (3), Faraday's Law for the magnetic field sensor may be rewritten

$$-\frac{d}{dt}\ B\ A_{eq} = \int \mathbf{E} \cdot dl = V_o$$

or, more simply,

$$\dot{B}\ A_{eq} = V_o, \qquad (7)$$

the transfer function of the B-Dot sensor.

The electric and magnetic fields which occur in a traveling electromagnetic wave are related to each other and to the power transmitted by the wave. The relationship is described by the Poynting Vector S, which is a measure of the power transmitted:

$$S = \frac{1}{\mu} E \times B$$

The Poynting vector and its constituents are illustrated in Figure 3. The magnitude of the vector is the product of the electric and magnetic fields, which represents the intensity of the traveling wave. It is usually given in terms of power per unit area, eg, w/m^2. The direction of the vector is the direction of propagation. If a wave is traveling through a surface of area A, the power transmitted through the area is:

$$P = S \cdot dA = SA$$

Then the power density or intensity of the wave is

$$S = P/A, \quad w/m^2$$

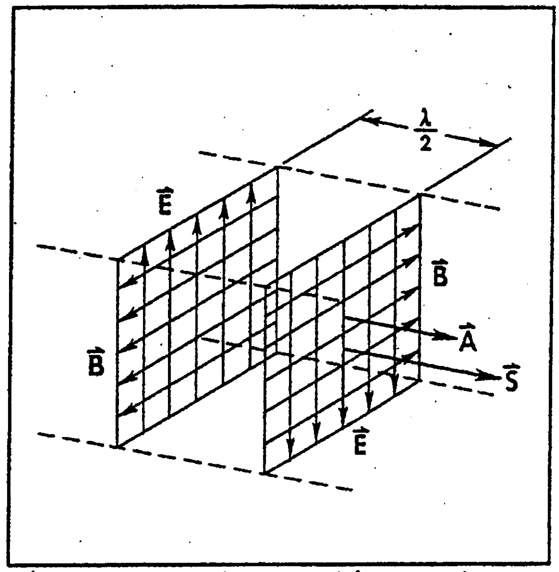


Figure 3. The Poynting Vector

For an electromagnetic wave propagating in free space, the magnitude of the Poynting vector is the power density:

$$S = EB/\mu_0 = P/A.$$

where S, E and B are the average values of the power density, electric field strength and magnetic induction. E and B vary sinusoidally, so

$$E = E_p/\sqrt{2} \qquad \text{and} \qquad B = B_p/\sqrt{2}$$

Then

$$S = E_pB_p/2\mu_0 = P/A. \tag{8}$$

This expression can be solved for either E_p or B_p using $E_p = cB_p$:

$$E_p = \sqrt{2\mu_0 cS} \qquad \text{and} \qquad B_p = \sqrt{2\mu_o S/c}.$$

We can measure E_p and/or B_p with a D-dot and/or a B-dot sensor, and calculate the power density in the wave.

We have shown that both electric and magnetic field sensors measure the first time derivative of the field strength by sensing the flux of the field through the sensor area. The voltage output of the sensor is the field flux density (field intensity) multiplied by the equivalent area (sensitivity) of the sensing element. For example, suppose it is necessary to record an electric field waveform which is expected to be an exponential step with a magnitude of about 50 kV/m and a risetime on the order of .5 microseconds (see Figure 4).

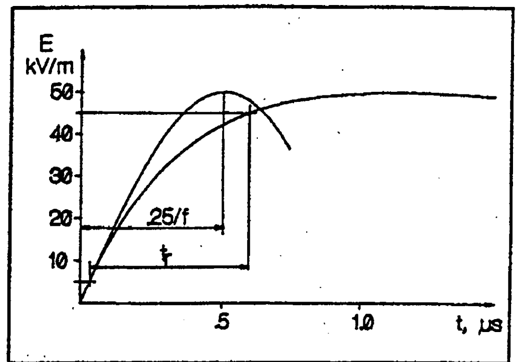


Figure 4. Electric field exponential step

The risetime can be converted to an equivalent frequency (the frequency having the same maximum rate of change as the step) by the simple formula (developed in Reference 1) $f = .25/t_r$, where f is the equivalent frequency and t_r is the 10-90% risetime of the step:

$$f = .25/t_r = \frac{.25}{.5\text{x}10^{-6}\ \text{sec}} = .5\text{x}10^{6}/\text{sec} = 500\ \text{khz}$$

The equivalent cw waveform, a quarter sine step, may be represented by

$$D(t) = \epsilon_o E(t) = \epsilon_o E_p \sin 2\pi f t$$

so that

$$\dot{D} = 2\pi f \epsilon_o E_p \text{Cos}\ 2\pi f t$$

and

$$\dot{D}_p = 2\pi f \epsilon_o E_p$$

$$= 2\pi\ \frac{.5\text{x}10^6}{\text{sec}}\ 8.85\text{x}10^{-12}\ \frac{\text{coul}}{\text{V-m}}\ 50\text{x}10^3\ \frac{\text{V}}{\text{m}} = 1.39\ \frac{\text{A}}{\text{m}^2}$$

Suppose now that we are using a D-dot sensor with an equivalent area of $10^{-2}\ \text{m}^2$ and a balanced, 100 ohm output. From the D-Dot sensor transfer function (Equation (6)) we have:

$$V_o = R\ A_{eq}\ \dot{D} = 100\ \text{ohm}\ (10^{-2}\ \text{m}^2)\ 1.39\ \text{A/m}^2 = 1.39\ \text{V}$$

If we desire more or less output, we simply use a larger or smaller sensor. Choosing the size sensor is thus a matter of calculating the maximum value of D-Dot and matching the desired voltage output with the equivalent area using the transfer function.

The principle illustrated in the foregoing applies to magnetic field sensors also, but there seems to be a wider and more confusing array of parameters used to specify the field intensity, e.g. gammas, Gauss, Gilberts, Henries, lines (Maxwells)/cm^2, Oersteds, Teslas, Webers/m^2, and doubtless some other enterprising graduate student has done or will define another unit and name it after himself. Webers/m^2, which are actually volt-seconds/m^2, work best with the B-Dot sensor transfer function, so some conversion relationships are listed here:

Units of magnetic flux density (B):

$$1\ \frac{\text{weber}}{\text{m}^2} = 1\ \text{tesla} = 10^4\ \text{gauss} = 10^4\ \frac{\text{lines}}{\text{cm}^2} = 10^5\ \text{gamma}$$

Units of magnetic induction (H):

$$1\ \frac{\text{amp-turn}}{\text{m}} = \frac{4\pi}{10^3}\ \text{oersted} = \frac{4\pi}{10^3}\ \frac{\text{gilbert}}{\text{cm}}$$

Units of magnetic permeability (μ):

$$1\ \frac{\text{weber}}{\text{amp-m}} = 1\ \frac{\text{v-sec}}{\text{amp-m}} = 1\ \frac{\text{henry}}{\text{m}} = 1\ \frac{\text{ohm-sec}}{\text{m}}$$

For example, suppose it is necessary to record a magnetic field waveform which is expected to be an exponential step with a magnitude of about 5 Oersteds and a risetime on the order of .5 microseconds (see Figure 5). We first convert the magnetic field strength to magnetic induction by multiplying it by the permeability:

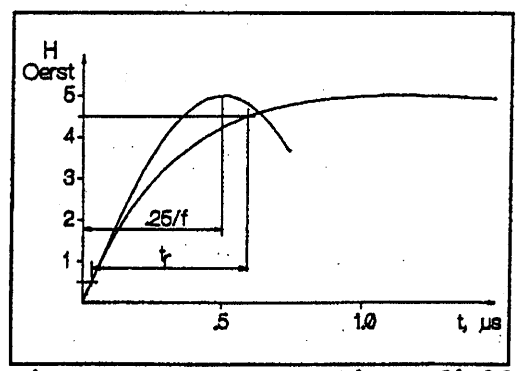


Figure 5. Magnetic field exponential step

$$B_p = \mu_o H_p$$

$$= 4\pi \text{x} 10^{-7}\ \frac{\text{v-sec}}{\text{amp-m}}\ 5\ \text{oersteds}\ \frac{10^3\ \text{amp}}{4\pi\ \text{oersted m}}$$

$$= 5\times10^{-4}\ \frac{\text{v-sec}}{m^2}$$

The risetime can be converted to an equivalent frequency just as before:

$$f = .25/t_r = \frac{.25}{.5\times10^{-6}\ \text{sec}} = .5\times10^{6}/\text{sec} = 500\ \text{khz}$$

The maximum time rate of change is obtained as with electric field, i.e., multiplying the equivalent cw frequency by the maximum change in the field strength:

$$\dot{B}_p = 2\pi f B_p$$

$$= 2\pi\ \frac{.5\times10^{6}}{\text{sec}}\ 5\times10^{-4}\ \frac{\text{V-sec}}{m^2} = 1.57\times10^{3}\ \frac{V}{m^2}$$

Suppose now that we want to select an appropriate B-dot sensor to produce a signal of 1 to 2 volts. From the B-dot sensor transfer function (Equation (7)) we have:

$$V_o = A_{eq}\,\dot{B} = 2\ V$$

$$A_{eq} = \frac{2\ V}{1.57\times10^{3}\ V/m^2} = 1.27\times10^{-3}\ m^2$$

A common equivalent area among PRODYN B-Dot sensors is $10^{-3}\ m^2$. With this equivalent area,

$$V_o = A_{eq}\,\dot{B} = 10^{-3}\ m^2\ 1.57\times10^{3}\ V/m^2 = 1.57\ V$$

We can simultaneously measure the electric and magnetic fields in a continuous EM wave using a D-dot sensor with $A_{eq} = A_D$ and output impedance R_D, and a B-dot sensor with $A_{eq} = A_B$. Assuming the wave has a peak electric field E_p, a peak magnetic field B_p and frequency f, we would expect sensor output voltages V_D and V_B given by Equations (6) and (7) respectively:

$$V_D = A_D\, R_D\, 2\pi f\ \epsilon_o E_p \quad \text{and} \quad V_B = A_B\, 2\pi f\ B_p$$

The ratio of the sensor output voltages is:

$$\frac{V_B}{V_D} = \frac{A_B\ 2\pi f\ B_p}{A_D\ R_D\ 2\pi f\ \epsilon_o E_p} = \frac{A_B\ B_p}{A_D\ R_D\ \epsilon_o E_p}$$

One of the most profound results of Maxwell's electromagnetic theory is the relationship of the electric and magnetic fields in a traveling electromagnetic wave ($E = cB$) and the resultant description of the properties of the medium in which the wave travels ($c^2\ \mu_o\ \epsilon_o = 1$). The field strength relationship can be rewritten as $B_p/E_p = 1/c$ and substituted in the ratio expression, giving

$$\frac{V_B}{V_D} = \frac{A_B}{A_D\ R_D\ c\epsilon_o}$$

The medium property relationship can be rewritten as $c\mu_o = 1/c\epsilon_o = Z_o$ (the impedance of free space, 377 Ω) and substituted in the ratio expression, giving

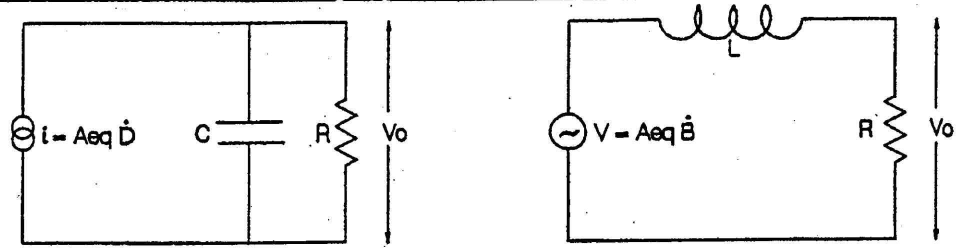
$$\frac{V_B}{V_D} = \frac{A_B\ Z_o}{A_D\ R_D} \qquad (9)$$

A typical PRODYN free-field D-dot sensor has a balanced output impedance of 100 Ω (two opposing 50 Ω impedances to ground). If the two sensors have the same equivalent area ($A_B = A_D$), the sensor output ratio becomes

$$\frac{V_B}{V_D} = \frac{377\ \Omega}{100\ \Omega} = 3.77$$

In other words, for a continuous electromagnetic wave in free space, the B-dot sensor voltage output will be 3.77 times that of a D-dot sensor with the same equivalent area and 100 Ω output impedance.

Rigorous developments of the transfer functions of B-Dot and D-Dot sensors are given in References 2 and 3 respectively, and summarized in Figure 6. In these developments, the sensing element is shown in an equivalent circuit with an inductance (B-Dot) or capacitance (D-Dot) and a load impedance (see Figure 6). A differential equation representing current is written in the time domain and then transformed into the frequency domain by Laplace transforms, where it is solved for the output. In both cases, the output is represented as a quotient whose numerator is the flux•area product we have been



In the time domain:

$$i = A_{eq}\dot{D} = C\frac{dV}{dt} + \frac{V}{R} = C\dot{V} + \frac{V}{R} \qquad V = A_{eq}\dot{B} = L\frac{di}{dt} + Ri = L\dot{i} + Ri$$

Transforming to the frequency domain with the Laplace transform:

$$[F(t)] = F(s) \qquad s = j\omega$$

$$[\dot{F}(t)] = sF(s) \qquad |s| = |j\omega| = \omega$$

$$A_{eq}\ sD(s) = C\ sV(s) + V(s)/R \qquad A_{eq}\ sB(s) = L\ si(s) + R\ i(s)$$

$$V(s) = \frac{A_{eq}\ sD(s)\ R}{RCs + 1} \qquad V(s) = \frac{A_{eq}\ sB(s)}{sL/R + 1}$$

Low frequency transfer functions (The differentiating mode):

for $\omega RC << 1$, for $\omega L/R << 1$,

$$V(s) = A_{eq}\ sD(s)\ R \qquad V(s) = A_{eq}\ sB(s)$$

or, transforming back to the time domain,

$$V(t) = A_{eq}\ \dot{D} \qquad V(t) = A_{eq}\ \dot{B}$$

High frequency transfer functions (The self integrating mode):

for $\omega RC >> 1$, for $\omega L/R >> 1$,

$$V(s) = A_{eq}\ D(s)/C \qquad V(s) = A_{eq}\ B(s)\ R/L$$

or, transforming back to the time domain,

$$V(t) = A_{eq}\ D/C \quad V(t) = A_{eq}\ B\ R/L$$

Figure 6. Sensor transfer function development

discussing and whose denominator contains two terms, one being dependent on both the sensor characteristics and the signal frequency (sRC for the D-Dot sensor or sL/R for the B-Dot sensor) and the other being unity.

The transfer functions given above are obtained by neglecting the frequency dependent term and are only valid when the signal frequency is low enough that the frequency dependent term is small with respect to unity. When this condition is met, the sensor is said to be operating in the "differentiating mode", i.e., the output is proportional to the derivative of the field intensity (see Figure 7). PRODYN's D-Dot and B-Dot sensors are designed with very small RC and L/R "time constants" such that sRC and sL/R are small compared to unity in the high megahertz to low gigahertz frequency range.

When the signal frequency is very high, the frequency dependent terms sRC and sL/R become large with respect to unity and the high frequency transfer function becomes valid. When this condition prevails, the sensor is said to be operating in the "self integrating" mode, i.e., the output is proportional to the integral of the derivative of the field intensity.

When the signal frequency is high enough that the low frequency transfer function is not valid, but not high enough to validate the high frequency transfer function, the sensor is said to be operating in the "transitional mode". In this mode, the full transfer function must be used without benefit of the simplifications which apply to the low and high frequency modes. The transition frequency is defined as that frequency for which the frequency dependent term equals unity. The complete transfer functions are represented by the graph in Figure 7, which shows that the actual transfer function approaches the low and high frequency transfer functions asymptotically. A more complete discussion of the frequency response of differentiating sensors is given in Reference 1, wherein the error factors given in Figure 7 are derived.

REFERENCES

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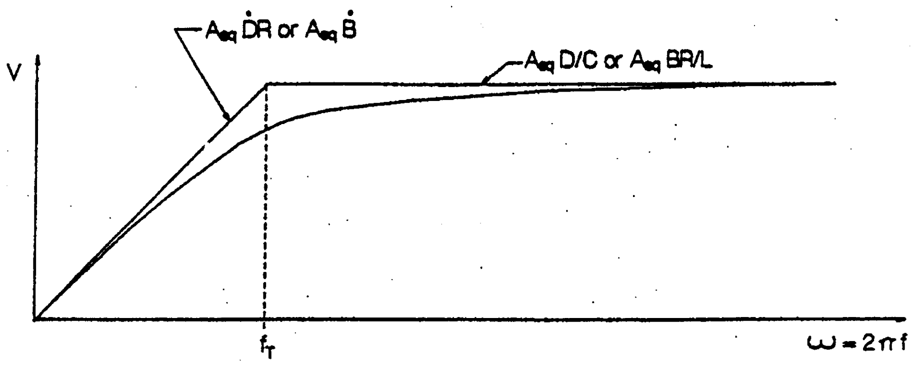

$$e_1 = \frac{A_{eq}DR - V}{A_{eq}\dot{D}R} \quad \text{or} \quad \frac{A_{eq}B - V}{A_{eq}\dot{B}}$$

$$e_2 = \frac{A_{eq}D/C - V}{A_{eq}D/C} \quad \text{or} \quad \frac{A_{eq}B/L - V}{A_{eq}B/L}$$

Errors for various values of $a = f/f_T$:

a	1/10	1/7	1/4	1/3	1/2	1	2	3	4	7	10
e_1	.005	.01	.03	.05	.106	.293					
e_2						.293	.106	.05	.03	.01	.005

Figure 7. Operating modes of electric and magnetic field sensors