PRODYN APPLICATION NOTE 512
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PASSIVE INTEGRATORS

The Real Meaning of an RC Circuit

Introduction

This document discusses how an RC circuit is used as an Integrator.

Theory of the RC Circuit

The basic theory of the RC circuit is developed by considering the RC circuit shown in figure 1.

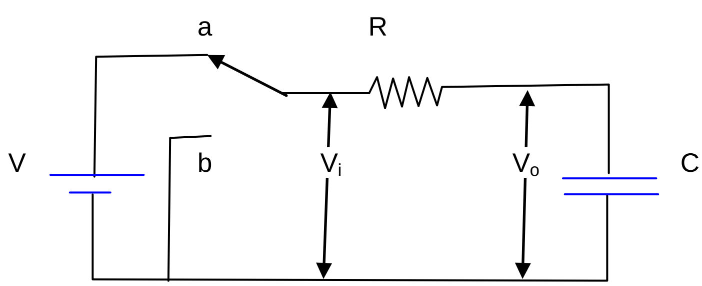


Figure 1. RC Circuit.

Let the switch be thrown to a at t = 0 and to b at t = T. Ideally, the voltage applied to the RC ciruit (V_i) during the time interval 0 to T is a rectangular pulse of height V_p and width T, as shown in figure 2a. The real pulse has a startup delay, a finite rise time t_r, overshoot and oscillation about V_p as shown in figure 2b.

The voltage across the capacitor (V_o) during the interval $0 < t < T$ is

$$V_o(t) = V_p\,(1 - e^{-t/RC}) \qquad (1)$$

Where

V = battery voltage, volts
R = resistance of circuit, ohms (V/A)
C = capacitance of circuit, Farads (A-sec/V)
T = width of the input pulse

The voltage across the capacitor (V_o) as a function of time is shown in figure 2c.

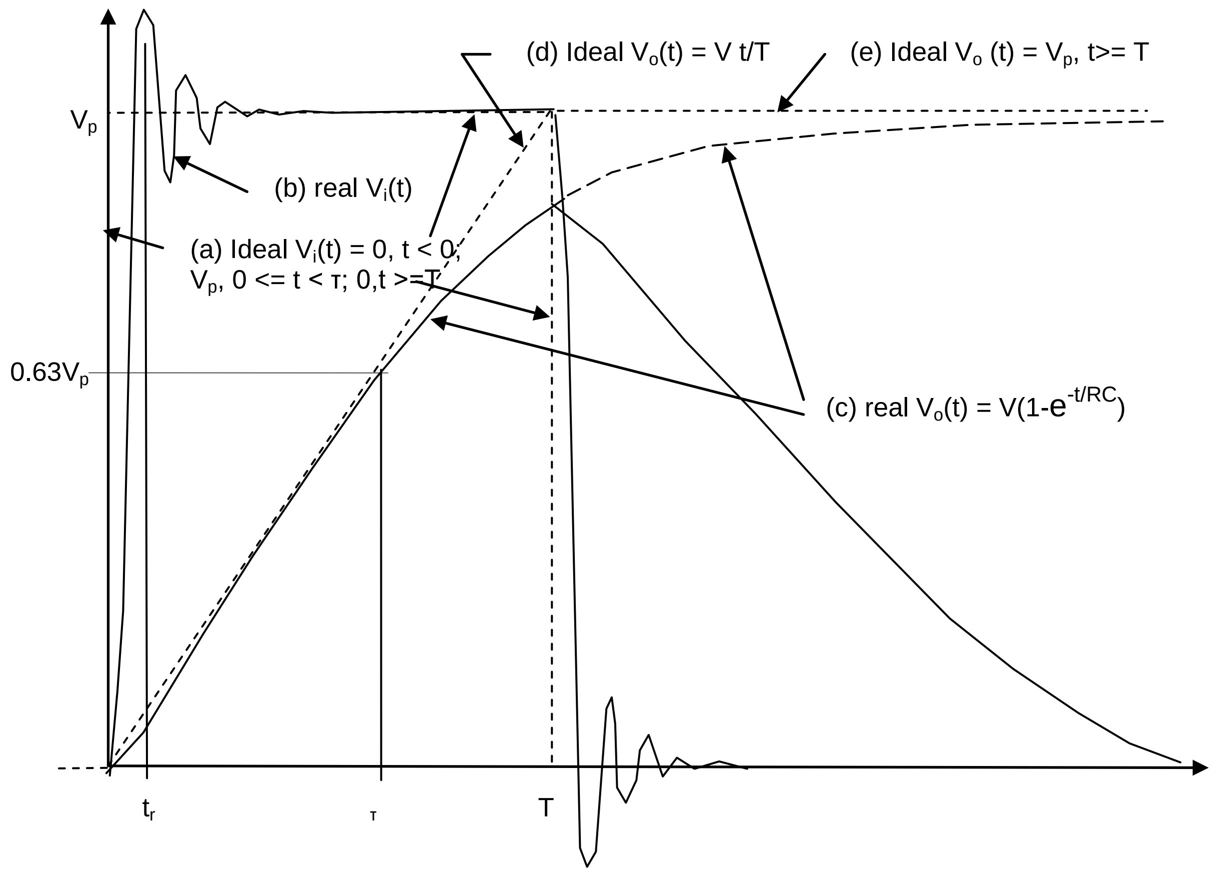


Figure 2. Ideal step function.

The integral of Vp over the interval 0 < t < T is VT/2

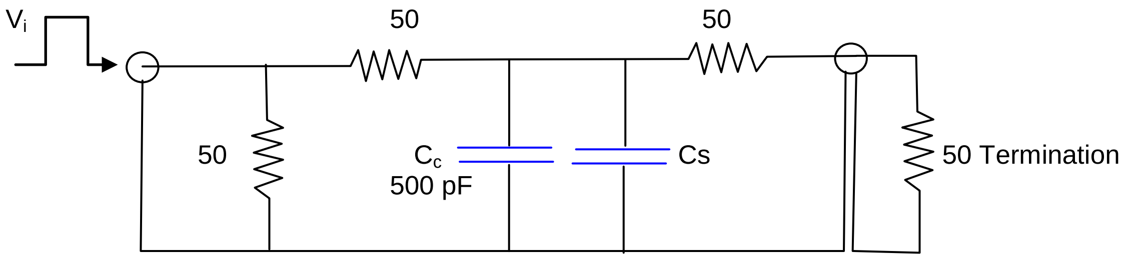
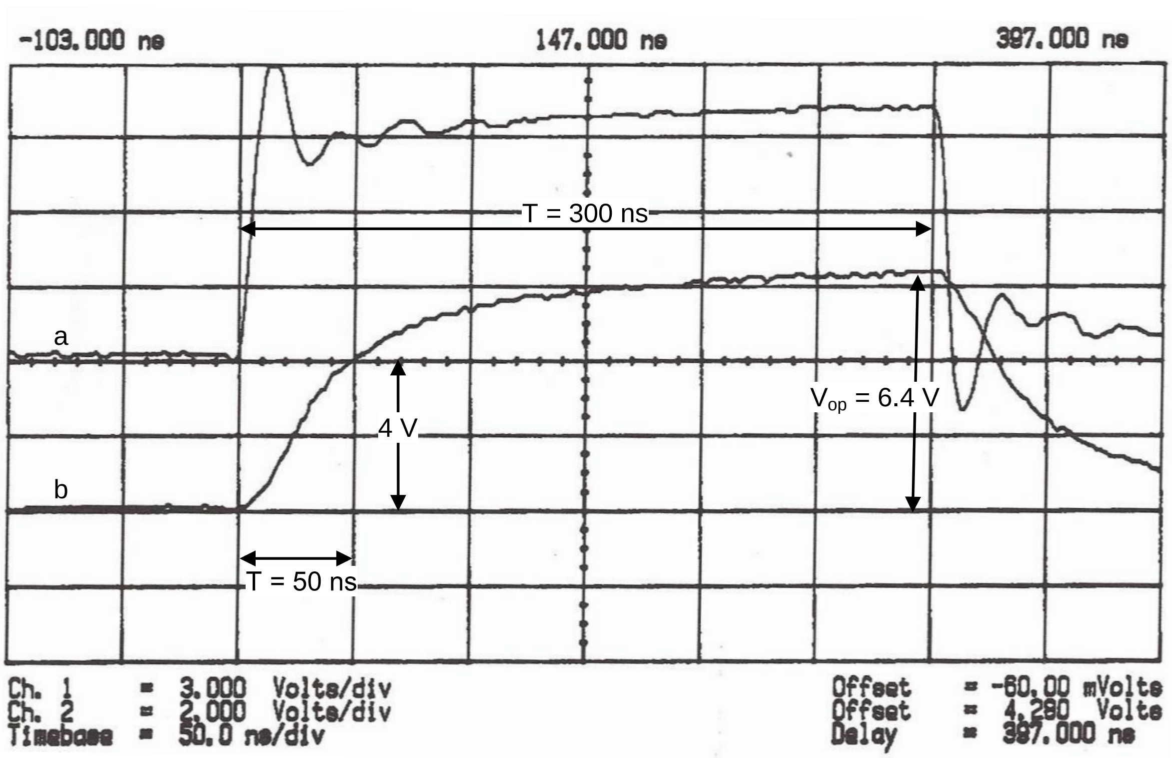


Figure 3. Passive integrator using RC Circuit.

Figure 3 shows the schematic diagram of a PRODYN PA-05 Passive integrator. It is intended to have an RC time constant of 0.05 microseconds (μs) which is 50 nanoseconds (ns) and to have its output terminated in 50 ohms (volts/amp, V/A). It is shown having a square pulse input, the pulse having a nominal amplitude (height) of 10 V and a duration (width) of 300 ns. The input pulse is shown in figure 4a.

The output pulse is shown in figure 4b. It is a good representation of the antiderivative or integral of the input pulse. An analysis of the output pulse follows.



The time constant can be calculated from the data by noting that Vo(t) will be 63% of its final value at t = τ. We observe from the data that:

V_{op} = 6.4 Volts; $V_o(\tau) = 0.63 \times 6.3\ V = 4.03\ V$; $\tau \rightarrow V_o = 4\ V = 50\ ns = 0.05\ \mu s$

The RC time constant of the circuit is R times C, or $\tau = RC$. Then

$$C = \tau/R = 50 \times 10^{-6}\ s / 50\ V/A = 10^{-6}\ A\text{-}s/V = 1\mu F = 1000\ pF$$

There is some stray capacity between the leads and the housing and the gremlins in the integrator. The effective capacity of the integrator is

$$C = C_c + C_s \qquad (2)$$

Where

C = total capacitance of the RC integrator circuit, picofarads (pF)

Cc = capacitance of installed capacitor, pF

Cs = stray capacitance of packaging and leads, pF

Rewriting equation (2),

$$C_s = C - C_c = 1000\ pF - 500\ pF = 500\ pF$$