# PRODYN APPLICATION NOTE 1103A

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# Functional Testing

# The Next Best Thing to Calibration

Introduction

This document discusses functional testing vs calibration of differentiating electromagnetic sensors, including "D-dot" and "B-dot" sensors that respond to the first time derivative of the incident electric and magnetic fields.

Theory of Transverse Electromagnetic Radiation

The basic theory of transverse electromagnetic radiation is discussed in PAN 895, Pages 2-4.

Testing vs Calibration

The term "calibration" is not appropriate, because there is no other standard except another magnetic field sensor that can measure the incident field accurately. It is only possible to compare the output of the subject sensor to the output of another magnetic sensor, preferably an identical one. These sensors themselves are the standards. While they cannot be calibrated, they can be functionally tested.

A major challenge over the last four decades has been to find a source of transverse electromagnetic (TEM) waves that was consistent with its theoretical fields to the extent that it could be used to calibrate EMP sensors. Large EMP simulators such as ALECS, ARES and TRESTLE were built at Kirtland Air Force Base in Albuquerque. They were mapped with electric and magnetic sensors like the subject sensor. The sellers of TEM cells of less than room size dimensions provide maps of field strength versus position in the working volume. These maps are generally made with some form of electric or magnetic field sensor.

Testing done in the 1970's when this class of sensor was developed was done in large simulators with the distance between conducting elements h of the order of 5 to 20 meters, many times the largest dimension of the item under test. In the early days of EMP testing, pulsers were based on mercury wetted reed relays and physical charge lines instead of solid state switching devices and electronic timing devices. Sensor outputs from the early models of the subject sensor were integrated digitally or with mechanical planimeters. Sensor outputs were found to be accurate to within the experimental error of the test setup, typically 4 to 7 percent. The equivalent area is controlled within 1 percent, so these sensors were used to map or "calibrate" the simulators.

The distinction between a sensor and a transducer and the lack of need for calibration is discussed in more detail on page 7 of PAN 895.

Smaller EMP simulators were designed for functional testing of sensors. These included a 15 foot high by 3 foot diameter conical simulator and a smaller conical-elliptical simulator. One advantage of these simulators over the plethora of small parallel plate simulators and TEM cells currently available is that they have radial symmetry about the axis of propagation of the EM wave and eliminate one dimension of distortion of the wave. They also are large enough that the ratio of simulator working height, *h* to the height of the sensor, *d* is larger than that for most small parallel plate simulators and TEM cells.

The presence of the sensor perturbs the electric field generated by the simulator. The perturbation has the effect of reducing the electric field and thence the magnetic field (See PAN 895, page 3) the sensor senses. The degree of perturbation depends on the ratio of simulator working height, *h* to the height of the sensor, *d*—the higher the simulator working height, the less a sensor of a given size perturbs the fields.

The interaction between the sensor and the simulator also exhibits frequency dependence. Location of the sensor in a simulator of limiting size is critical. Capacitative coupling between sensor and simulator becomes significant—small changes in the location of the sensor with respect to the simulator can have much larger effects on the effective electric and magnetic fields. The typical EM sensor would not see these effects so profoundly in a large simulator such as ALECS, ARES, or TRESLE, where h ranges from 5 to 25 meters and h/d ranges from 10 to 50.

The foregoing means that small parallel plate simulators or TEM cells cannot be used to "calibrate" EM sensors because the incident fields are affected more by the presence of the sensor than the response of the sensor is affected by inaccuracies in its manufacture. Comparison of one kind of sensor to another only results in a comparison of the perturbations caused by different kinds. The best functional tests performed in the 1970s showed that the sensors were more accurate than the test setups.

One of the most frequently asked questions we get at PRODYN is "What is the calibration factor that gives B, D or I from the output of a B-dot, D-dot or I-dot sensor?" The answer is that there is no such factor, because *the output of this class of sensors (differentiating sensors) is proportional to the first time derivative of the input to the sensor, not to the input itself*. This is an advantage when the phenomenon of concern responds to the first time derivative of the input, which is often the case. Another advantage differentiating sensors have is that fields with large rates of change provide larger outputs from smaller sensors.

There is an exception, when the input is sinusoidal, that arises from the fact that the derivative of the sine function is the cosine function. This case will be discussed after the general case wherein the input is not sinusoidal.

The inherent lack of proportionality between fields and sensor outputs is a major reason why differentiating sensors cannot be “calibrated”. While they cannot be calibrated, they can be functionally checked. For the benefit of the technologist who needs a number that describes the sensitivity of this class of sensors to electric or magnetic fields, the following discussion should be helpful.

While the foregoing applies to electric and magnetic field sensors and to current sensors, we will use magnetic field (B-dot) sensors to illustrate how the value of the field strength is obtained from the output of a differentiating sensor when the field steps from zero to some level.

The process is idealized in figure 1. The magnetic field, or more correctly, the magnetic induction, B, is measured in Teslas, or volt-seconds per square meter. As shown in figure 1(a), it rises from zero to $B_p$ in time $t_m$. The rise is shown as a ramp function, with a rounded transition from zero rate of increase to the maximum rate of increase as the signal "takes hold" and a rounded transition from the maximum rate of increase to the flat value of $B_p$. The field is rising fastest at $t_{mr}$ near the middle of the "step".

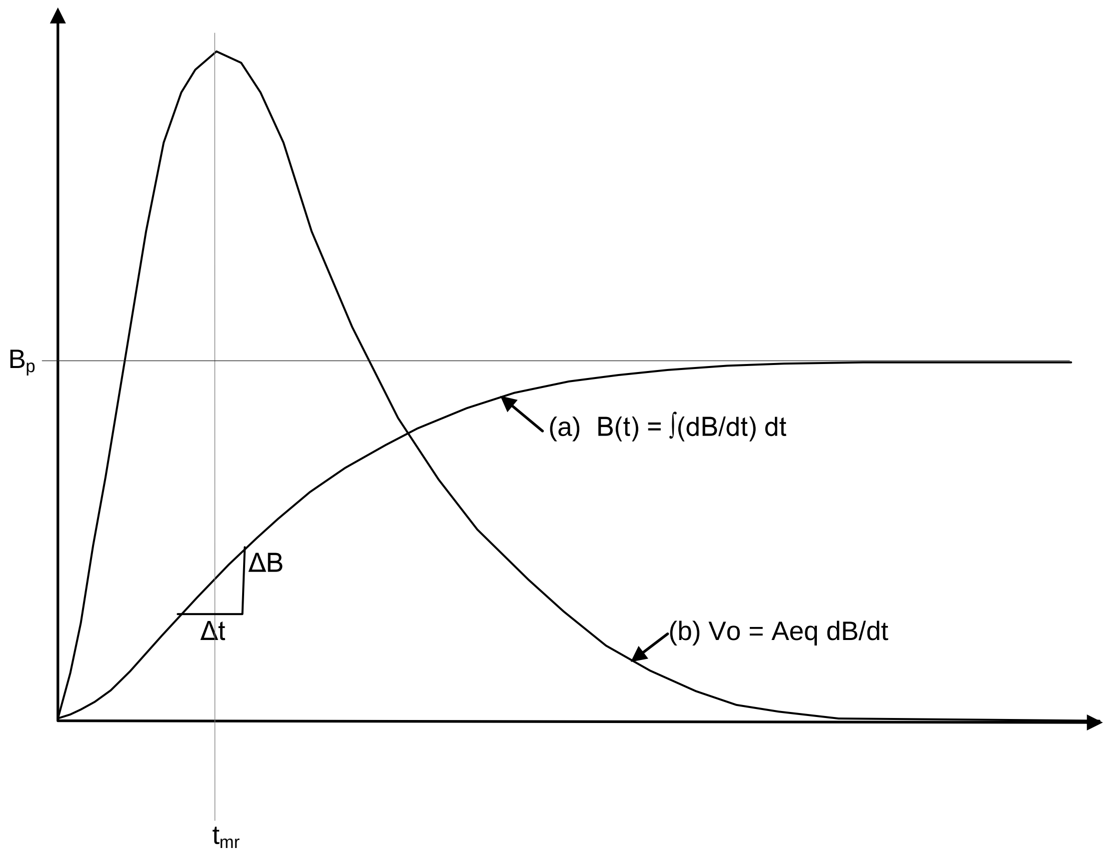


**Figure 1. Step function magnetic field and its derivative.**

The output of the B-dot sensor is proportional to the first time derivative of the magnetic induction B(t), that is:

B-dot = dB/dt ~ ΔB/Δt

and, rewriting the transfer function of the sensor,

$V_o = A_{eq}\ dB/dt$, or $dB/dt = V_o/A_{eq}$

The output of the B-dot sensor is measured in volts that are proportional to first time derivative of the magnetic induction, which has units of volts per square meter. The constant of proportionality between the sensor output $V_o$ and the first time derivative of the magnetic induction dB/dt is the equivalent area $A_{eq}$ of the sensor. This is the calibration factor, but it is applied to the first time derivative of the magnetic induction, not to the magnetic induction itself.

The sensor output resulting from the input of figure 1(a) is shown in figure 1(b). It is a single pulse that rises as the rate of increase in B increases to a maximum value, and then decreases as the rate of increase in B decreases to zero when B reaches its peak value.

A reasonable estimate of the maximum value of dB/dt is ΔB/ Δt, which is obtained by measuring the change in B over the time interval during which B is changing most rapidly, just before the middle of the ramp function in figure 1(a). This is the maximum output we expect from the sensor.

The magnetic field B(t) is obtained by integrating the sensor output with respect to time. In mathematical shorthand, $B(t) = \int(dB/dt)\,dt$. If the sensor output has been digitized, the integral can be obtained easily by multiplying the numerical values of ΔB/Δt by Δt over the time interval 0 to t. As the number of discrete points increases, ΔB/Δt becomes a better estimate of dB/dt and (ΔB/Δt) Δt becomes a better estimate of $B = \int dB = 1/A_{eq} * V_o dt$. Ideally, the function is identical to that in figure 1(a).

A discussion of the meaning of ideal is in order at this point. Consider an ideal step function as shown in figure 2. The function (figure 2a) steps from zero to $B_p$ at $t = t_o$ in zero time. The rise time is zero and the derivative (figure 2b) is infinite. With real waveforms, we can set the time base of an oscilloscope so as to make the function appear to our finite eye as a pure step function, but we cannot measure the infinite derivative. We can, however, expand the waveform by reducing the time base and see how close we are to the ideal. We will then see the waveforms shown in figure 1.

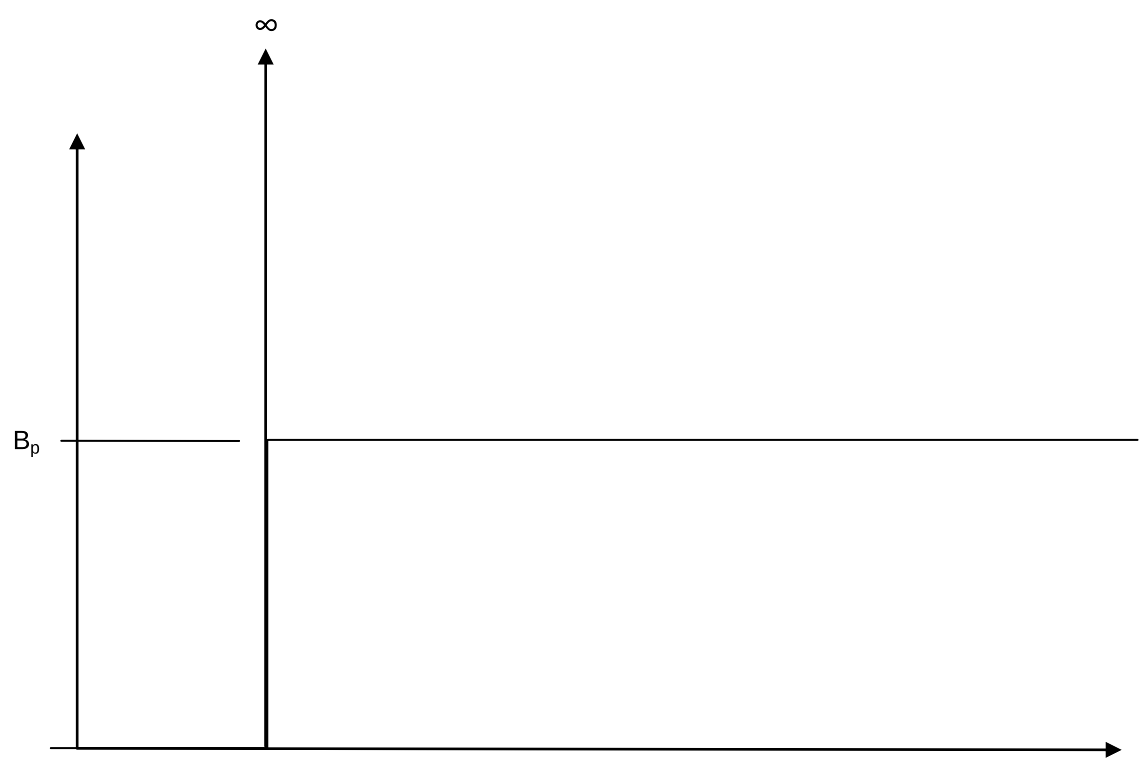


**Figure 2. Ideal step function.**

When the magnetic field forms a single pulse such as that in figure 3(a), it rises from zero to $B_p$ in time $t_r$. The rise is shown as a steep ramp function, with a rounded transition from zero rate of increase to the maximum rate of increase as the signal "takes hold" and a rounded transition from the maximum rate of increase to the maximum value of $B_p$. The field is rising fastest near the middle of the "step". The field then falls off, usually more slowly than it rose, to zero.

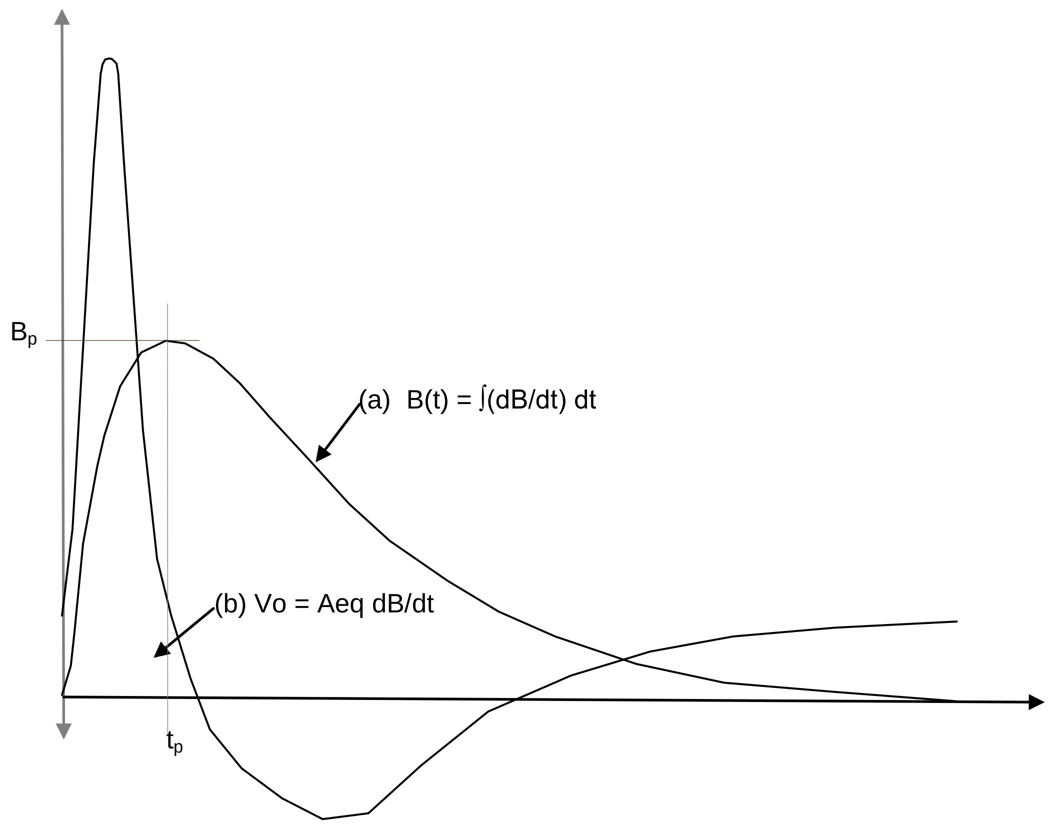


**Figure 3. Single pulse magnetic field and its derivative.**

Note that the output of the sensor is at its peak when the rate of change of the magnetic field is fastest and zero when the field is at its peak. This is the expected result from calculus. The sensor output goes negative when the magnetic field starts decreasing (the rate of change of the magnetic field becomes negative and returns to zero when the matnetic field returns to the constant value of zero. Again, this is the expected result from calculus.

The value of the field strength is obtained from the output of the differentiating sensor the same way as before, by integrating with respect to time. Again, the resulting function is ideally identical to that in figure 3(a).

Sinusoidal field functions make it possible to integrate the output by simply dividing by the angular frequency of the signal. We will use an electric field for this case. The process is idealized in figure 4. The electric field E which in free space is the electric displacement D divided by the permittivity constant $\varepsilon_{o,}$ is measured in volts per meter. The electric displacement is measured in Amp-seconds per meter$^2$. As shown in figure 3(a), it is a continuous sinusoidal function with amplitude $D_p$ and frequency f.

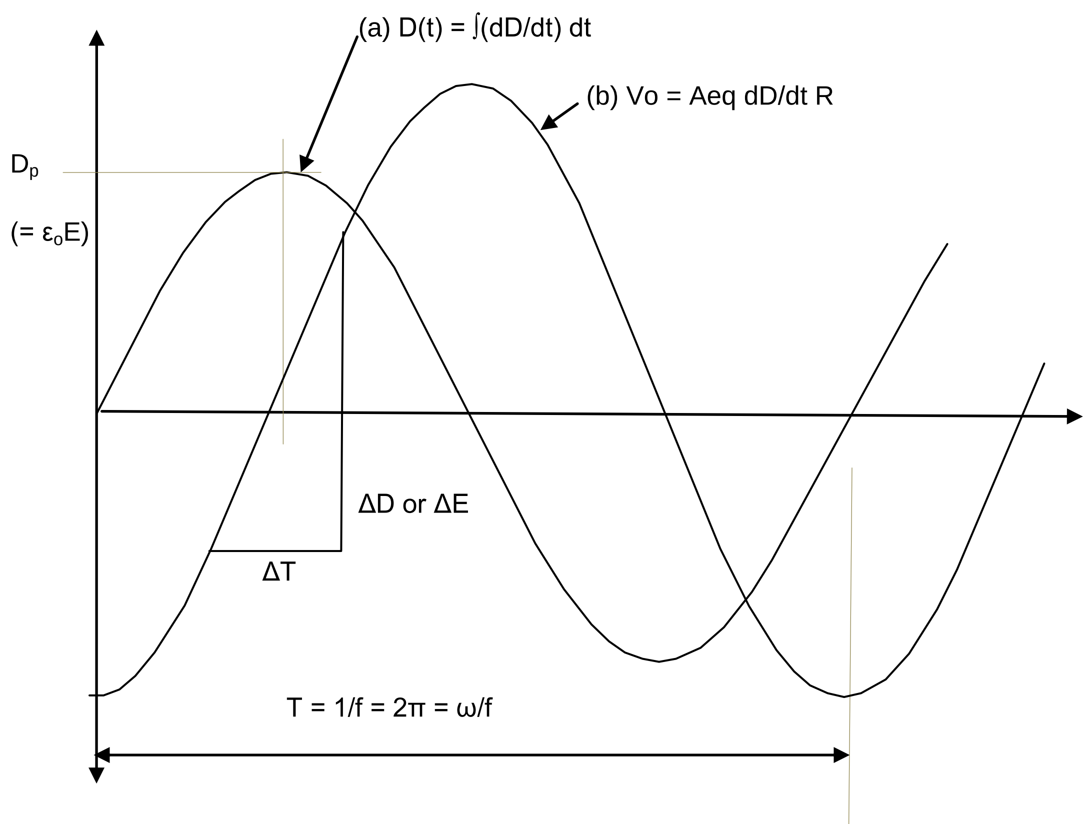


**Figure 4. Sinusoidal electric field and its derivative.**

The output of the D-dot sensor is proportional to the first time derivative of the electric displacement D(t), that is:

D-dot = dD/dt ~ ΔD/Δt

and, rewriting the transfer function of the sensor,

$V_o = R\ A_{eq}\ dD/dt$, or $dD/dt = V_o/R/A_{eq}$

then, recalling that $D = \varepsilon_o\ E$,

$dE/dt = V_o/R/A_{eq}\ /\varepsilon_o$

The output of the D-dot sensor is measured in volts that are proportional to the first time derivative of electric displacement dD/dt, which has units of coulombs per square meter or amp-seconds per square meter. The constant of proportionality between the sensor output and the first time derivative of the electric field dE/dt (= dD/dt $\varepsilon_o$) is the permittivity constant $\varepsilon_o$. The constant of proportionality between the sensor output $V_o$ and the first time derivative of the electric field dE/dt is the the product of the output impedance R and the equivalent area $A_{eq}$ of the sensor, and the permittivity constant $\varepsilon_o$. This is the calibration factor, but it is applied to the first time derivative of the electric field, dE/dt, not to the electric field itself.

The sensor output resulting from the input of figure 4(a) is shown in figure 4(b). It is a cosine function with amplitude $V_{op}$ and frequency f, where:

$E_p = 1/2\pi f \, (dE/dt)_p = V_{op}/2\pi f \, R \, A_{eq} \, \varepsilon_o$

In the sinusoidal or CW case, the peak value of the input field can be obtained from the peak value of the sensor output, the constant of being proportionality being the product of the general constant R $A_{eq}$ $\varepsilon_o$ and $2\pi f$, which is constant if the frequency is constant when the measurement is made, as in the CW case. Note also that the output of the sensor increases as the frequency increases.

A reasonable estimate of the maximum value of $\Delta D/\Delta t$ or $\Delta E/\Delta t$ is obtained by measuring the change in D or E over the time interval during which D or E is changing most rapidly. This is usually taken as the T/6 duration when D or E is going from +0.5 ($D_p$ or $E_p$) to -0.5 ($D_p$ or $E_p$), as shown in figure 3(a). This is the maximum output we expect from the sensor.

The integrated sensor output is obtained by integrating the sensor output with respect to time. If the sensor output has been digitized, the integral can be obtained easily by multiplying the numerical values of $\Delta D/\Delta t$ by $\Delta t$ over the time interval 0 to t. As the number of discrete points increases, $\Delta D/\Delta t$ becomes a better estimate of dD/dt and $(\Delta D/\Delta t)\Delta t$ becomes a better estimate of $D = \int dD = 1/A_{eq} * V_o dt$. Ideally, the function is identical to that in figure 4(a).

The foregoing mathematical exercise reduces to the following practical guideline: If the sensor waveform looks like the derivative of the field, the sensor is probably working, and in the absence of some gremlin, is reporting the first time derivative of the field as it should. So in fact, while functional testing is falls a little short of calibration in the sense that its results are not traceable to what we have come to regard as authority, it is the next best thing to calibration in that we can take comfort in the assurance that the sensor is operating as it was intended and is telling us the truth.